An element e of a ring R is called an idempotent if $e^2 = e$. An idempotent e in R is said to be primitive if there are no two non-zero idempotents f and g in R such that $e = f + g$ and $fg = gf = 0$.

D. Dolzan [2] studied the set M of primitive idempotents of finite rings and proved that M is closed under multiplication if and only if R is a direct sum of local rings. In this talk, we consider the structure of M and some multiplicatively closed subsets of M. Then we generalize the result of Dolzan to more general rings. Let R denote a ring with $1 \neq 0$ and J denote its Jacobson radical. A ring R is called a semilocal ring if R/J is a semisimple Artinian ring. A semiperfect ring R is a semilocal ring with the property that every idempotent in R/J can be lifted to R. It is well known that R is semiperfect if and only if every finitely generated left R-module has a projective cover. We first we prove the following:

Theorem: Let R be a semiperfect ring with identity and M be the set of all primitive idempotents and zero in R. Then the set M is closed under multiplication if and only if R is a direct sum of local rings.

Let [M] denote the set $\{eR \mid e \in M\}$, that is, [M] is the set of right ideals of the form eR for some primitive idempotent e and the ideal 0. We consider the structure of rings R when [M] is closed under multiplication and prove the following:

Theorem: Let R be a semiperfect ring with identity. Then the set [M] is closed under multiplication if and only if R is a finite direct sum of matrix rings over local rings.

Reference

[1] C. W. Curtis and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras*, Wiley-Interscience,New York,1962.

[2] D. Dolzan, *Multiplicative sets of idempotents in a finite ring*, J. Algebra 304 (2006), no. 1, 271--277.