Hamiltonian-Connected Line Graphs with Given Degree Sums

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Abstract

In 1984, Bauer proposed the problems of determining best possible sufficient conditions on the vertex degrees of a simple graph (or a simple bipartite graph, or a simple triangle-free graph, respectively) G to ensure that its line graph $L(G)$ is hamiltonian. We investigate the problems of determining best possible sufficient conditions on the vertex degrees of a simple graph G to ensure that its line graph $L(G)$ is hamiltonian-connected, and prove the following.
(i) Let G be a simple graph on n vertices and a, b be real numbers with $0 < a \leq 1$. There exist an integer $N(a, b)$ and a finite family $\mathcal{F}(a, b)$ such that if $n \geq N(a, b)$ and if $d_G(u) + d_G(v) \geq an + b$ for any $u, v \in V(G)$ with $uv \notin E(G)$, then either $L(G)$ is hamiltonian-connected, or $\kappa(L(G)) \leq 2$, or G can be contracted to a member in $\mathcal{F}(a, b)$.
(ii) Let G be a simple graph on n vertices. If $d_G(u) + d_G(v) \geq \frac{n}{4} - 2$ for any $u, v \in V(G)$ with $uv \notin E(G)$, then for sufficiently large n, either $L(G)$ is hamiltonian-connected, or $\kappa(L(G)) \leq 2$, or G can be contracted to W_8, the Wagner graph.
(iii) Let G be a simple triangle-free (or bipartite) graph on n vertices. If $d_G(u) + d_G(v) \geq \frac{n}{8}$ for any $u, v \in V(G)$ with $uv \notin E(G)$, then for sufficiently large n, either $L(G)$ is hamiltonian-connected, or $\kappa(L(G)) \leq 2$, or G can be contracted to W_8, the Wagner graph.

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