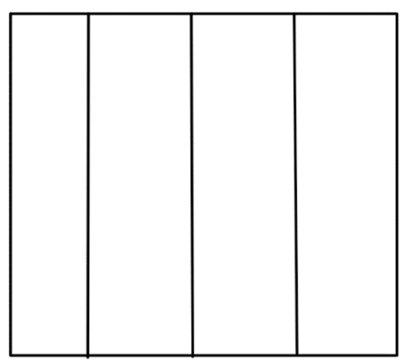
David Orcutt
MA 399
Dr. Hisa Tsutsui
19 June 2018

Maximized Pentagon within a Square

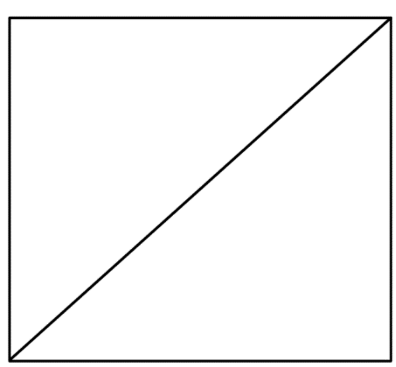
A pentagon can be formed from a square piece of paper by creating diagonal lines connecting points at the one-fifth, two-fifths and one half-marks.

Paper Division

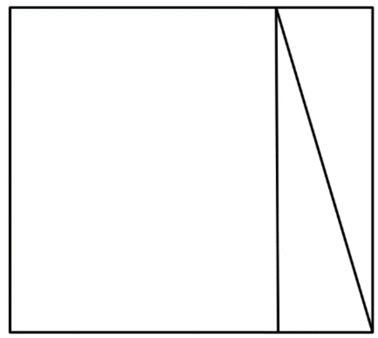
In the same manner a paper can be divided into thirds, a paper can be divided into fifths. To begin, fold the paper into fourths and unfold.



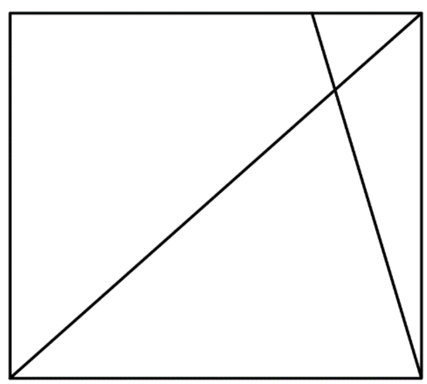
Fold diagonally.



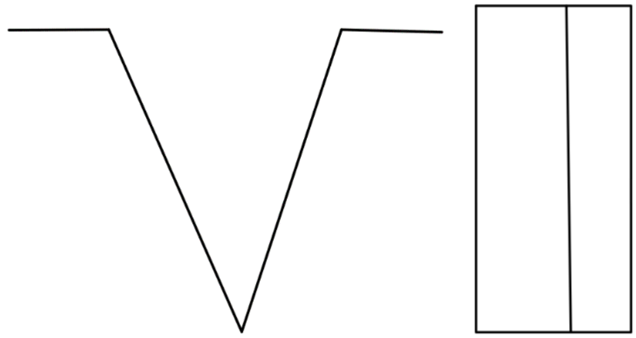
Also fold diagonally, but in the opposite direction, in the right one-fourth section.



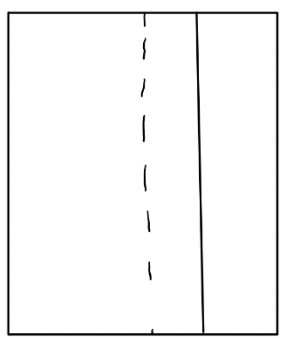
The point at which the two diagonal lines intersect is one fifth from the right and top edge. Make a horizontal and vertical fold parallel to the top and right edges respectively at this point.



To mirror the vertical fold, fold the paper in half -- bringing the left and right edges together -- and fold along the opposite side using the already folded side as a guide. Viewed from the side with both one fifth sections folded in opposite directions, the piece of paper should look like a ‘V’ with truncated serifs.

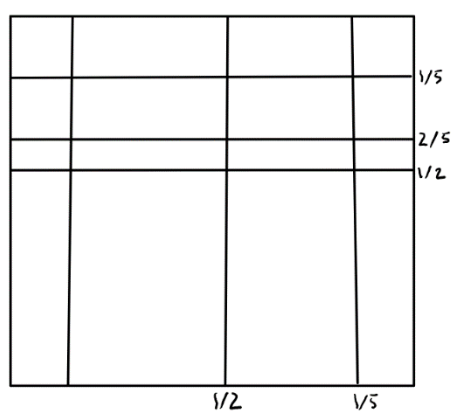


Unfold the paper and mirror the one-fifth section on the top to make a two-fifths section on the top.



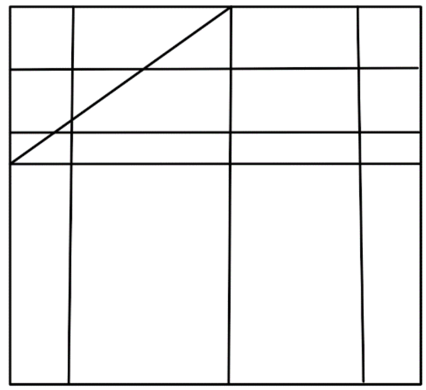
Vertical 1/2 Line

The completed grid:

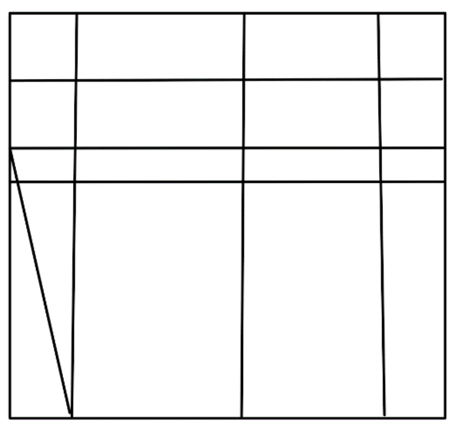


Creating the Pentagon

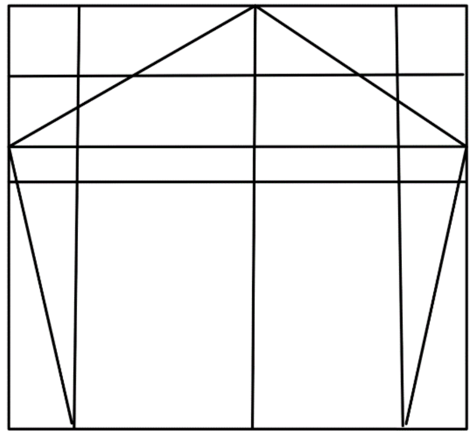
Make a fold that connects the point where the vertical one-half line intersects the top to the point where the two-fifths line intersects the right edge.



Connect the point where the two-fifths line intersects the right edge to the point where the right one-fifths line intersects the bottom edge.

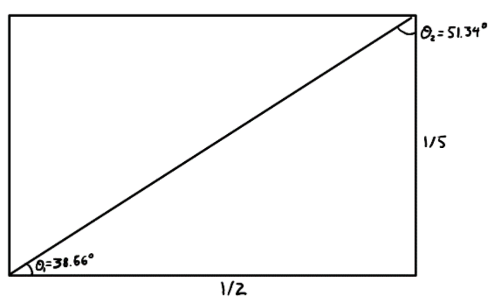


Repeat the previous two steps for the right side.

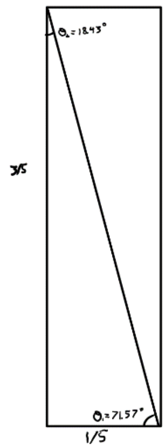


Explanation of Geometry

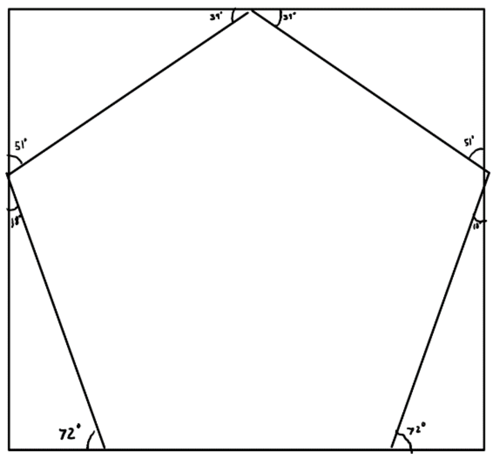
We shall assume the length of each side of the square is one. If we consider the upper left quadrant, we can see a right triangle with opposite side length of two-fifths and adjacent side length of one-half is formed. Since the theta between the hypotenuse and adjacent can be found by taking the arctangent of the opposite over the adjacent, the angle is angle 38.66 degrees. The theta between the opposite and hypotenuse is thus 51.34 degrees.



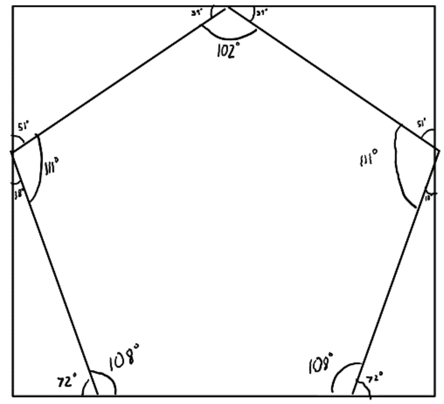
For the bottom left quadrant, we can make the same calculations, except the opposite is three-fifths and the adjacent is one-fifth, yielding the respective angles of 71.57 and 18.43 degrees.



These calculations can be reflected onto the right side.



Combining the previous two calculations, we can see the angle at the top become 102 degrees, leaving the side angles at 111 degrees. A pentagon should be comprised of 108 degree angles between its sides as shown on the bottom two angles. Since this calculation is imprecise, the mathematical interest behind this specific division into halves and fifths may be minimal, however, the principle of diving a paper to create a perfectly maximized pentagon may still be valuable and, regardless, provides a means of easily creating a pentagon from a square piece of paper.


102°
51°
111°
18°
51°
111°
108°
72°
108°
72°